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On the Design of Sustainable and Fair PAYG Pension Systems When Cohort Sizes Change.

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Editorial

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Abstract

In this paper we deal with the question how to make PAYG pension systems financially resistant to fluctuating fertility rates. We present two pension schemes that lead to a permanently balanced budget but differ in the mixture of changes in the contribution rates and replacement rates they require in order to achieve this result. After analyzing the variations in the central parameters (both over time and across generations) for each of the schemes we discuss which consequences they have with regard to intergenerational burden sharing and fairness. In particular we are interested in how a generation is affected by changes in the size of proceeding and succeeding cohorts. We introduce a "proportionality measure"(defined as the ratio of relative inputs to relative outputs) that can be used as an indicator to study this impact. We show that the schemes have quite different implications concerning how past and future cohorts influence the proportionality measure. Finally we discuss how suitable the formulas are to be implemented in either traditional PAYG or in "notional defined contribution" (NDC) systems.

Keywords: Pension System; Demographic Change; Intergenerational Fairness
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1 Introduction

Pay-as-you-go (PAYG) pension systems have recently come under severe pressure from demographic developments that hamper fiscal consolidation and might jeopardize long-run fiscal sustainability. The demographic challenge has various components. First, most industrialized countries had to observe a constant decline in fertility rates where the baby boom of the 60's was followed by a decrease in birth rates in the 70's. Second, medical progress and changes in general lifestyle have lead to a steady increase in longevity where it is now predicted that life expectancy will increase by one year every 6 to 8 years. Reinforcing this development, the retirement age has decreased in many countries (mostly as a consequence of generous early retirement programs) such that the true span that the average person spends in pension has grown even more strongly. Finally, macroeconomic developments and changes in labor market participation are a permanent source of fluctuations in the employment level and thus also in the contribution base of pension systems. It is forecasted that the combined impact of these changes will lead to a considerable increase in the old-age dependency ratio (for Austria, e.g., from 22.9% in 2000 to 40.7 in 2030). All of these fluctuations have caused doubts about the sustainability of current PAYG pension systems and have given rise to various efforts to reform these systems, either via more piecewise, "parametric" reforms or via the introduction of "notional defined contribution" (NDC) systems like in Sweden (cf., Disney 1999, Palmer 2000, Lindbeck and Persson 2003).

In this paper we deal with the first component of these demographic changes, the decline in fertility rates that is often said to be the main force behind the increase in old-age dependency ratios and the expected financial problems of the PAYG pension system. In Figure 1 we illustrate the size of birth cohorts for Austria from 1955 to 2000. One can clearly see the baby boom and baby bust cycle from the 60's to the 70's, the general downward trend and the non-negligible fluctuations on a year-to-year basis. We will present a stylized model of a PAYG system that allows us to study how the system could be made resistant to such changes in fertility rates. In particular we will present two pension schemes that have the property that they lead to a balanced budget of the system in every period for any conceivable pattern of cohort sizes. Thus as a first result of the paper we show that bold statements concerning the inadaptability of PAYG systems with respect to demographic fluctuations are unjustified. In fact, there exist a variety of modifications and demographic adjustment factors that are simple and transparent enough to be de facto incorporated into existing PAYG systems.

`Insert Figure 1 about here`

For both schemes the annual changes in contribution rates and pension levels depend on a comparison between the size of the average active cohort and the average retired cohort (scheme A) or a comparison between the own size of a cohort and respective average cohort sizes (scheme B). In both cases a weighting parameter determines whether the adjustment to demographic fluctuations is primarily achieved by changing the contribution rates or by changing the pension levels. Scheme A resembles in fact the "sustainability factor" that was recently introduced into the German pension system (cf. KNFSS 2003; Börsch-Supan, Reil-Held & Wilke, 2003).

In later parts of the paper we have a closer look at these sustainable schemes and we discuss the consequences of choosing different weights. In particular we deal with the crucial issue which pension scheme and which weight seems to be most appropriate to deal with fluctuations in fertility rates. In order to answer this question we compare them with respect to three dimensions: fluctuations in the central parameters, implications for intergenerational fairness and aspects of implementability.

First we analyze which variation in contribution rates and pension levels are associated with various schemes, both over time and between generations. Not surprisingly we find out that schemes that divide the adjustment to demographic changes rather evenly between contributors and pensioners avoid excessive fluctuations in contribution rates and pension levels. Such in-between weights are also most appropriate to achieve certain target levels for the two crucial magnitudes, i.e. a maximum contribution rate and a minimum pension level at the same time. In fact, such considerations seem to have been the main motivation behind the weight that was chosen for the German system. "The point of departure for the reform proposals was to achieve a politically pre-defined contribution target. [...] The sustainability factor with a weighting α of 0.25 corresponds most closely to the targeted contribution rates. [...] [and it] will achieve the contribution rate targets of 20 percent in 2020 and 22 percent in 2030." (Börsch-Supan, Reil-Held & Wilke, 2003, 20ff.).

It is, however, unclear whether these criteria are the most relevant and should be given the highest priority when deciding on the weight of a pension scheme. Large changes in contribution rates or pension levels could be justified and regarded as "fair" if they are related to differences in economic or social behavior between generations. One might, e.g., argue that cohorts with smaller numbers of descendents can be expected to shoulder a larger part of the demographic burden, both because they are partly responsible for the drop in the size of birth cohorts and because they are in a better (financial) situation to make good for the shortfall through private provisions. We will thus also look at the properties of different pensions schemes under the perspective of intergenerational fairness. This issue is often neglected and sometimes even dismissed in current debates[1] which are basically centered around questions of sustainability. Although it is understandable that the tense financial situation of today's systems makes this point of view the number-one political priority one should still try not to lose sight of what the proposed reform measures imply for the intergenerational distribution of demographic burdens and whether this distribution could be regarded as intergenerationally fair.

We present a simple "proportionality measure" that can be used as an indicator to study issues of intergenerational fairness. The measure is defined as the sum of relative inputs a cohort contributes to the system divided by the sum of relative outputs it receives from it. We can study how the proportionality measure fluctuates under each of the proposed pension schemes. Furthermore we analyze how strong the measure for one

[1] Cf. on this, e.g., Breyer 2000, Börsch-Supan, 2003, Sakai 2003.

cohort reacts to changes in the size of some other cohort. We show, e.g., that schemes that hold the contribution rate constant are more "forward looking" (the proportionality measure reacts stronger to the size of successor than to the size of predecessor generations) while frameworks with fixed pension levels have a more "backward looking" character. Forward-looking schemes thus have the property that the proportionality measure is to a larger degree influenced by the size of cohorts for which a generation is at least partly responsible. One could therefore argue that schemes with rather fixed contribution rates are more in line with principles of intergenerational fairness and responsibility. Even if one dismisses these conclusions as too strong it is nevertheless important to be aware that every chosen pension scheme has in fact some underlying structure of "cohort size dependencies".

Finally, the pension schemes differ with respect to their implementability. We argue that schemes with varying contribution rates are harder to reconcile with a NDC system than schemes with fixed rates. We also discuss how collective pension schemes (i.e., schemes that treat all members of a cohort alike) could be modified by introducing individual-based factors thereby bringing them more in line with principles of *intra*generational fairness.

The paper is organized as follows. In the next section we will present a simple balance sheet example of a PAYG system that allows us to discuss the main concepts used in the paper. The model is introduced in section 3 and the two financially sustainable pension schemes are presented in section 4. Section 5 then compares these schemes with regard to what they imply for fluctuations in the main parameters, to issues of intergenerational fairness and to their implementablity. Section 6 concludes.

2 A Simple Example

Table 1 presents the balance sheet of a stylized PAYG pension system. Each generation is of equal size ($N = 100$), lives for three periods each and receives pension benefits for one period. The generations are denoted by the first period of their working life (so generation t starts to work in period t) and the rows of Table 1 indicate for each generation the contribution rates τ they face in each of the first three periods (in light shading) and also the (relative) pension level q they are awarded in the last period of their life (in darker shading).[2] The latter is defined as the ratio of the pension p_t to average current income $\bar{w}_t$, i.e. $q_t = \frac{p_t}{\bar{w}_t}$.

`Insert Table 1 about here`

The income of the pension system in period t is given by the product of the average contribution rate $\bar{\tau}_t$, the average wage rate $\bar{w}_t$ and the total number of workers L_t. The total outflows in period t are given by the product of the average pension level $\bar{q}_t$ (in this example this is equal to q_t since we have only one generation of pensioners), the average

[2] We will introduce more precise notation in the next section.

wage rate $\bar{w}_t$ and the total number of pensioners R_t. A balanced budget thus requires that $\bar{\tau}_t L_t = \bar{q}_t R_t$. The pension system in Table 1 is balanced in every period which can be seen from the fact that the column sum is always zero. In period t, e.g., the system takes in 60 wage units ($[0.2 \times 100 + 0.1 \times 100 + 0.3 \times 100] * \bar{w}_t$)[3] and pays out also 60 wage units ($[0.6 \times 100] * \bar{w}_t$).

While the fiscal properties (the "sustainability") of the PAYG system can be read from the column sum of the balance sheet in Table 1, the engagement with question of intergenerational distribution and fairness requires a look at the rows. In the last columns of Table 1 we have calculated for each generation the sum of relative inputs (the contribution rates) and relative outcomes (the pension levels) and in the last column we report the ratio of these two magnitudes. Inspecting these figures shows that the picture here is less balanced than it was for the period budgets in the last row of the table. The sum of relative inputs and also their ratio is different across the cohorts and generations $t-1$ and t have ratios of 1.2 and 0.86, respectively, while all other generations have a ratio of 1. Most people would argue that the pension system in Table 1 is unfair since it does not treat generations equally, favoring generation $t-1$ and penalizing generation t. By the same token most people would prefer a pension system where the contribution rate is 0.2 and the relative pension level is 0.6 in every period and for each generation.

This is in a nutshell the measure of intergenerational fairness (we will call it the "proportionality measure" PM_t) which we will frequently use in later parts of the paper to study distributional consequences of various pension schemes and pension formulas. We will discuss later the concepts underlying this proportionality measure and the circumstances under which it is useful to employ. There we will deal with cases where the population size differs between cohorts (thus causing intergenerational heterogeneity).

3 The Model

3.1 Set-up

Generalizing the example of the last section we assume that each generation works for G periods and receives a pension for H periods. G and H are assumed to be constant across generations.[4] The average member of a cohort earns real wages $w_{1,t}$, $w_{2,t+1}$, ..., $w_{G,t+G-1}$ and pension payments $p_{1,t+G}$, $p_{2,t+G+1}$,..., $p_{H,t+G+H-1}$. The first subindex in these expression stands for the period g of the working career (or the period h of the retirement phase) which a generation lives through while the second subindex refers to the point in time when this payment is received. In each working period the representative member of a cohort faces a contribution rate $\tau_{g,t}$. We denote a generation by its first working period, which means that in period t generation t earns $w_{1,t}$, generation $t-1$

[3] We assume here, as in the rest of the paper, that all workers that are active in period t earn the same wage, i.e. $\bar{w}_t = w_t$.

[4] Without doubt increases in life expectany are an important source of demographic fluctuations that are also responsible for the current financial pressure of PAYG pensions systems. The treatment of this issue in the current paper would, however, go beyond its scope.

earns $w_{2,t}$ etc. The size of generation t is denoted by N_t.[5] The cohort growth rate n_{t+1} is given by:

$$N_{t+1} = N_t(1 + n_{t+1}) \tag{1}$$

The total number of workers L_t and the total number of retired persons R_t at time t is thus given by:

$$L_t = \sum_{g=1}^{G} N_{t-g+1} \tag{2}$$

$$R_t = \sum_{h=1}^{H} N_{t-G-h+1} \tag{3}$$

and the average number of workers (pensioners) is denoted by: $\bar{L}_t = \frac{1}{G}L_t$ $(\bar{R}_t = \frac{1}{H}R_t)$.

The average wage $\bar{w}_t$ and the average pension $\bar{p}_t$ in period t can be written as:

$$\bar{w}_t = \frac{\sum_{g=1}^{G} N_{t-g+1} w_{g,t}}{L_t} \tag{4}$$

$$\bar{p}_t = \frac{\sum_{h=1}^{H} N_{t-G-h+1} p_{h,t}}{R_t} \tag{5}$$

In addition we define the (relative) pension level $q_{h,t}$ by:

$$q_{h,t} = \frac{p_{h,t}}{\bar{w}_t} \tag{6}$$

The magnitude $q_{h,t}$ thus indicates which fraction of average income $\bar{w}_t$ the representative member of a generation receives in his or her h'th year of retirement.

Finally we define the average contribution rate $\bar{\tau}_t$ and the average pension level $\bar{q}_t$ by:

$$\bar{\tau}_t = \frac{\sum_{g=1}^{G} N_{t-g+1} \tau_{g,t}}{L_t} \tag{7}$$

$$\bar{q}_t = \frac{\sum_{h=1}^{H} N_{t-G-h+1} q_{h,t}}{R_t} \tag{8}$$

This completes the basic structure of the model. In the rest of the paper we will abstract from seniority wages, i.e. we will assume that:

$$w_{g,t} = w_t, \forall g \tag{9}$$

It thus follows that $\bar{w}_t = w_t$ and that $\bar{q}_t = \frac{\bar{p}_t}{w_t}$. The growth rate of wages γ_{t+1} is given by:

$$w_{t+1} = w_t(1 + \gamma_{t+1}) \tag{10}$$

[5] We do not distinguish between population and labor force in this paper and thus N_t stands interchangeably for both magnitudes.

3.2 The Budget of the Pension System

Using assumption (9) and equations (2) to (8) the income IN_t, expenditures EX_t and the deficit D_t of the pension system in period t can be written as:

$$IN_t = \sum_{g=1}^{G} N_{t-g+1}\tau_{g,t}w_{g,t} = w_t\bar{\tau}_t L_t \tag{11}$$

$$EX_t = \sum_{h=1}^{H} N_{t-G-h+1}p_{h,t} = \bar{p}_t R_t = w_t\bar{q}_t R_t \tag{12}$$

$$D_t = EX_t - IN_t \tag{13}$$

The balanced budget condition (BBC) is given by $D_t = 0$ which implies the following relation:

$$\bar{\tau}_t L_t = \bar{q}_t R_t \tag{14}$$

3.3 Steady State Values

In this subsection we will look at the "demographic steady state" of this model economy, that is a state where the working population is constant: $N_t = N$, $\forall t$. From (2), (3) and (14) it follows that the BBC reduces to: $\bar{\tau}_t G = \bar{q}_t H$. The "reference values" (or "steady state values") $\widehat{\tau}$ and $\widehat{q}$ are given by values for the contribution rate and the pension level that are constant over time (i.e., $\tau_{s,t} = \widehat{\tau}$ and $q_{s,t} = \widehat{q}$, $\forall t, s$) and that fulfill the BBC (14). We thus have the steady state relation:

$$\widehat{\tau}G = \widehat{q}H \tag{15}$$

The values for $\widehat{\tau}$ and $\widehat{q}$ can be freely chosen as long as (15) fulfilled.[6] In the example of section 2 we have assumed, e.g., that $\widehat{\tau} = 0.2$ and $\widehat{q} = 0.6$. The preferences of a society will determine which combination of $\widehat{\tau}$ and $\widehat{q}$ is chosen. This might depend, inter alia, on the life-cycle patterns of needs and necessities, on political and electoral processes and on preferences concerning intertemporal substitution and risk-aversion. We will not delve further into this issue here and we will simply assume that some values for $\widehat{\tau}$ and $\widehat{q}$ are chosen that fulfill condition (15).

3.4 The Proportionality Measure

In this section we want to take up the discussion of the beginning about the question of intergenerational fairness and the availability and suitability of indicators that allow for reasonable statements and comparisons in this area. To this end we suggest the use of

[6] In principle also $\frac{G}{H}$ could be regarded as a political choice variable that can, e.g., be influenced by legislation about (early) retirement age etc. As said before, however, we abstract from these issues in this paper.

a proportionality index that captures — as we will argue — fairness aspects of pension systems and that is useful in judging how different schemes affect the distribution between generation. In particular we propose a proportionality measure PM_t that is defined as the sum of (undiscounted) pension levels allocated to generation t divided by the sum of contributions rates paid by this generation. Expressed in formal terms:

$$PM_t = \frac{\widetilde{q}_t}{\widetilde{\tau}_t}, \text{ where} \tag{16}$$

$$\widetilde{\tau}_t = \sum_{g=1}^{G} \tau_{g,t+g-1} \tag{17}$$

$$\widetilde{q}_t = \sum_{h=1}^{H} q_{h,t+G+h-1} \tag{18}$$

We do not claim that the proportionality measure is the best and only indicator for intergenerational fairness let alone for the quality and desirability of a pension scheme in general. We do believe, however, that the measure offers an interesting perspective on this highly controversial issue and that it is helpful in trying to figure out the distributional properties of various actual or proposed pension systems.

The measure PM_t is related to a number of existing theories and concepts. In order to justify its use we want to discuss two of them more extensively in the following.

3.4.1 The Proportionality Measure and Equity Theory

There exists a close connection between the proportionality measure and the "equity theory" that was developed by sociologists and social psychologists (Adams 1963, Walters et al., 1973) and later extended to the economic context (Selten 1978, Güth 1994).[7] According to the equity theory an allocation or distributional rule will be regarded as fair if for two persons A and B the ratio of input to outcome is identical.[8] The proportionality measure can be seen as the natural extension of this concept to an intergenerational context. A generation that has contributed a larger share of their wage income to the pension system should also be rewarded a larger fraction of average income when they are in pension.

Equity theory was criticized on various accounts, e.g. that one has to distinguish between inputs and outcomes that are under the control of the agent, that contexts with multiple inputs and outcomes might require more differentiated formulas and that existing differences in abilities might give rise to different preferred allocation rules serving

[7] A good survey of various fairness theories is Konow (2003). There it is also noted that equity theorists typically trace the origins of their approach to Aristotle's Nicomachean Ethics, where a theory of justice based on proportionality was introduced.

[8] This is normally expressed in the "equity formula": $\frac{O_A}{I_A} = \frac{O_B}{I_B}$. "Inputs $[I]$ are usually thought of as a participant's contributions to an exchange and outcomes $[O]$ as the consequences, potentially positive or negative, that a participant has incurred in this connection." (Konow 2003, 1211).

distributional objectives. These criticisms partly carry over to the intergenerational context. There might exists, e.g., differences between generations or in generational behavior that justify intergenerational differences in PM_t. As we will argue later (cf. section 5) this is especially relevant if cohorts differ with respect to the number of their descendants which alters their respective abilities and which could also be interpreted as a situation with multiple inputs (contributions *and* offsprings).

Furthermore, one has to be careful not to inappropriately "universalize" the proportionality measure as a criterion to judge the relative performance and quality of different pension schemes. Pension systems have other functions besides the creation of intergenerational fairness (or — as one might call it — "equitable intertemporal consumption smoothing"). Intergenerational risk-sharing, e.g., is another important property and role of pension systems (cf. Gordon & Varian 1988) and a deviation from strict proportionality might be justified by a reference to this task. In fact, a pension system that is proportional ex-ante will most likely lead to non-proportional outcomes ex-post once it also contains insurance elements that lead to redistribution to negatively affected individuals (or generations). Since in this paper we work in an environment of certainty we can disregard these issues and directly use the proportionality measure as an appropriate indicator of intergenerational fairness.

3.4.2 The Proportionality Measure and "Money's Worth Measures" of Pension Systems

There exist various "money's worth statistics" to evaluate the properties of existing pension schemes and the effects of proposed pension reforms (cf. Geanakoplos, Mitchell & Zeldes 1999, 84). Four widely used measures are the internal rate of return, the present value ratio, the net present value and the implicit tax rate (on the latter cf. Fenge & Werding 2003).

The proportionality measure is most closely related to the present value (or benefit/tax) ratio PVR_t which is defined as the ratio of the present value of benefits to the present value of contributions. In formal terms:

$$\begin{aligned} PVR_t &= \frac{\sum_{h=1}^{H} p_{h,t+G+h-1}\left(\prod_{j=2}^{G+h}\frac{1}{(1+\delta_{t+j-1})}\right)}{\tau_{1,t}w_t+\sum_{g=2}^{G}\tau_{g,t+g-1}w_{t+g-1}\left(\prod_{j=2}^{g}\frac{1}{(1+\delta_{t+j-1})}\right)} \\ &= \frac{w_t\sum_{h=1}^{H} q_{h,t+G+h-1}\left(\prod_{j=2}^{G+h}\frac{(1+\gamma_{t+j-1})}{(1+\delta_{t+j-1})}\right)}{w_t\left[\tau_{1,t}+\sum_{g=2}^{G}\tau_{g,t+g-1}\left(\prod_{j=2}^{g}\frac{(1+\gamma_{t+j-1})}{(1+\delta_{t+j-1})}\right)\right]} \end{aligned} \tag{19}$$

where (for $j>0$) δ_{t+j} is the discount rate between periods $(t+j-1)$ and $(t+j)$ and $\delta_t = 0$.[9]

The discount rate is often assumed to be equal to the (expected) real interest rate, i.e. $\delta_{t+j} = r$ for $j > 0$. Assuming a constant growth rate of wages γ equation (19) thus

[9]The second line in (19) uses (10).

simplifies to: $PVR_t = \frac{\sum_{h=1}^{H} q_{h,t+G+h-1}\left(\frac{1+\gamma}{1+r}\right)^{G+h-1}}{\sum_{g=1}^{G} \tau_{g,t+g-1}\left(\frac{1+\gamma}{1+r}\right)^{g-1}}$. For this case it is thus not in general true that $PM_t = PVR_t$ (only if γ and δ coincide). It is, however, not clear whether the real interest rate is in fact the appropriate discount rate to use in expression (19).

First we want to stress that we use the present value ratio to judge the intergenerational fairness of a pension scheme. Thus we need some intergenerational discount rate. There is a long debate about "discounting and intergenerational equity" and the most appropriate values to use.[10] In a different context Arrow (1995) argues for a social discount rate between 3% and 4% and similar or even lower values are proposed by other authors. These are figures that are considerably below the conventionally used measures for the real interest rate of around 6% and they are much closer to the growth rates of real wages.

In addition, however, it is questionable whether interest rates should be used as social discount factors in the context of intergenerational equity and fairness. Capital markets are in a way alien to the PAYG system and they do not appear at all in the set-up of our model. An individual will most likely use some interest rate as an opportunity rate to value future income streams but from the perspective of a "neutral observer" (or "intertemporal social planner") who wants to judge the intergenerational distribution this is less obvious. Finally we want to note that most of our main results (in particular how different pension schemes lead to different importance of cohort sizes) are qualitatively unaffected by the choice of the discount factor.

Summarizing the discussion of this section we believe that the proportionality measure is a good concept to study the intergenerational fairness of pension schemes. This is in particular true if one wants to assess the distributional consequences of various pension schemes in some stylized model economy. If a pension scheme shows strange properties in this artificial world one cannot expect that it will manage real-world demographic changes in a reasonable manner.

4 Methods to Deal With Fluctuations in the Size of Birth Cohorts

In this section we want to discuss how PAYG systems can be designed that remain balanced even in the wake of fluctuations in the cohort growth rate n_t. The challenge is to design a pension scheme that has desirable properties under these circumstances.

The clause "desirable properties" in the last sentence is important. Otherwise one could fill in the boxes of the PAYG balance sheet with any numbers (as long as $\tau < 1$ and $q > 0$) and this would constitute a pension scheme — most likely one with unbalanced budgets in every period and strange distributional results. In a way the design of a pension scheme can be compared to one of the most popular fields of "recreational mathematics": magic squares. For both purposes one has to fill in numbers into a matrix subject to certain constraints. For the simplest case of magic squares this amounts to inserting the

[10] An interesting discussion of matters of discounting in the area of pension systems can be found in Homburg (1990, chap. 4).

positive integers $1, 2, \ldots, n^2$ such that the sum of the n numbers in any horizontal, vertical, or main diagonal line is always the same number.[11] The design of a pension scheme is similar only that the "magic square" is of infinite dimension and that one has more possible and potentially conflicting constraints. A balanced budget constraint, e.g., means that the column sum has to be equal to zero in every period. Other constraints could for example be that the contribution rate should be constant for all generations at some time t (i.e. $\tau_{g,t} = \tau_t$, $\forall g$) or for each generation across time (i.e. $\tau_{g,t+g-1} = \tau_{1,t}$, $\forall g$) or the parallel constraint for the pension level (i.e. $q_{h,t} = q_t$, $\forall h$ and $q_{h,t+G+h-1} = q_{1,t+G}$, $\forall h$, respectively). Still other constraints could refer to the proportionality measure requiring, e.g., that $PM_t = 1$, $\forall t$ or that it is only a function of certain characteristics X_t of generation t, i.e. $PM_t = f(X_t)$. In general one would like to have a small set of simple and comprehensible formulas that determine the values for τ and q.

The actual choice between the different possibilities will depend on a variety of factors, including the preferences of society and policymakers, the availability of certain data, characteristics of the political system and other factors and concerns outside our model (e.g., the general tax system). In order to make an educated choice between the different possibilities it is, however, useful to know the effects and possible hidden properties of these schemes. This is the topic of the present section, where we will discuss two classes of pension schemes.[12] Since the requirement of sustainability is one of the most crucial demands in the current pension debate we will only consider pension systems that have a balanced budget in every period (i.e. condition (14) is fulfilled).

4.1 Case A: Comparing the average size of active and retired cohorts

First we want to look at a case where in each period all workers pay the same contribution rates (i.e. $\tau_{g,t} = \tau_t$, $\forall g$) and all pensioners receive identical pension levels ($q_{h,t} = q_t$, $\forall h$). Both magnitudes are determined by a comparison of the average cohort size of the working and the one of the retired population where a parameter α reflects the weight of adjustment borne by the two groups.

$$\tau^A_{g,t} = \tau^A_t = \widehat{\tau} \left[1 + (1 - \alpha) \frac{\bar{R}_t - \bar{L}_t}{\bar{L}_t} \right] \tag{20}$$

$$q^A_{h,t} = q^A_t = \hat{q} \left[1 + \alpha \frac{\bar{L}_t - \bar{R}_t}{\bar{R}_t} \right] \tag{21}$$

Using the definitions (2), (3) and (14) one can easily show that a pension system that is characterized by formulas (20) and (21) leads to a balanced budget in every period (see appendix). Since $\widehat{\tau}$, $\hat{q}$, L_t and R_t are known in period t these expressions unambiguously

[11] There exist many extensions and refinements of magic squares. Information can be found on the internet pages: http://mathworld.wolfram.com/MagicSquare.html or http://www.magic-squares.de.

[12] Alternative schemes can be found, e.g., in Lindbeck and Hassler 1997.

define the contribution rate and pension level for each generation.[13]. Equations (20) and (21) state how the adjustment to a drop in the average size of active cohorts $\bar{L}_t$ relative to the average size of retired cohorts $\bar{R}_t$ is borne by workers and pensioners. If $\alpha = 0$ the pension level is held constant at $\hat{q}$ for all time and for all cohorts and thus the full burden of adjustment to demographic fluctuations is shouldered by the working population via changes in the contribution rate. The reverse is true for $\alpha = 1$ where the contribution rate stays constant at $\hat{\tau}$ and solely the pension level is varied to achieve a balanced budget. In general α determines how the "demographic burden" is shared between contributors and pensioners. In fact, a scheme like this was introduced recently into the German pension system. The "sustainability factor" closely mirrors equations (20) and (21) and the relative adjustment weight was set equal to $\alpha = 0.25$.[14]

Using (20) and (21) and the definitions in (16) to (18) we can derive the proportionality measure for generation t:[15]

$$PM_t^A = \frac{\sum_{h=1}^{H}\left[(1-\alpha)\frac{G}{H} + \alpha\frac{\sum_{g=1}^{G} N_{t+G+h-g}}{\sum_{j=1}^{H} N_{t+h-j}}\right]}{\sum_{g=1}^{G}\left[\alpha + (1-\alpha)\frac{G}{H}\frac{\sum_{h=1}^{H} N_{t-G+g-h}}{\sum_{j=1}^{G} N_{t+g-j}}\right]} \tag{22}$$

In Table 2 we illustrate the working of this pension scheme for a situation with $G = 3$, $H = 1$, $\alpha = 0.5$ and where there is a drop in cohort size from $N_s = 100$ for $s < t$ to $N_s = 50$ for $s \geq t$.[16] In the example 6 generations are affected by this drop and they have a proportionality measure that is smaller than 1. The reduction is highest for the generation that immediately proceeds the first smaller cohort (the "parent generation") and it decreases with the distance from this cohort (in both directions). In general the size of cohort t affects the proportionality measure of the generations $(t-G-H+1)$ to $(t+G+H-1)$. Or equivalently, as can be seen directly from equation (22), the proportionality measure of generation t depends on the cohort sizes $N_{t-G-H+1}$ to $N_{t+G+H-1}$. We come back to this issue later when we will compare the properties of the various pension schemes.

`Insert Table 2 about here`

[13]Note that the pension can be easily calculated since $p_{h,t}^A = q_{h,t}^A w_t$.

[14]Cf. KNFSS, 2003 and Börsch-Supan, Reil-Held & Wilke (2003, 15). In the actual formula the adjustment factor is lagged by one period which is necessary due to problems of contemporeneous data availability. This is only a minor difference to the case we consider here and we could easily change our formulas to account for this lag. Also it depends on additional specificties of the German pension system which are not present in our model.

[15]The expressions for the sum of relative inputs $\widetilde{\tau}_t^A$ and the sum of relative outcomes $\widetilde{q}_t^A$ are stated in the appendix. For the calculation of PM_t^A we make use of (15).

[16]This example is, however, only used for illustrative purposes and all our results are of course valid for general patterns of N_t (as, e.g., the one for Austria from section 1).

4.2 Case B: Comparing the size of the own cohort to the average cohort size

Next we want to turn to a pension scheme that has the property that neither the contribution rate nor the pension level is necessarily identical for all cohorts at some point in time. It is characterized by the following two expressions:

$$\tau_{g,t}^{B} = \widehat{\tau}\left[1 + (1-\beta)\frac{\bar{R}_t - N_{t-g+1}}{N_{t-g+1}}\right] \tag{23}$$

$$q_{h,t}^{B} = \hat{q}\left[1 + \beta\frac{\bar{L}_t - N_{t-G-h+1}}{N_{t-G-h+1}}\right] \tag{24}$$

Equations (23) and (24) say in fact that the contribution rate paid by a working cohort g in a certain period does not depend on the relation of the average size of the pension cohorts ($\bar{R}_t$) to the average size of the working cohorts ($\bar{L}_t$) as in case A but rather on the size of $\bar{R}_t$ compared to the size of the *own* (working) cohort N_{t-g+1}. Similarly the pension level of cohort h depends on how the size of the own (retired) cohort $N_{t-G-h+1}$ deviates from the size of the average working cohort $\bar{L}_t$. Thus this scheme implies that each cohort is directly and fully affected by its own size. This differs from case A, where the use of average cohort sizes $\bar{L}_t$ and $\bar{R}_t$ leads to some intergenerational smoothing and where therefore all active cohorts and all retired cohorts "mutually insure" themselves against fluctuations in cohort size.

Assuming that the pension system is defined by the formulas (23) and (24) again leads to a budget that is balanced in every period (see appendix). The proportionality measure comes out as:

$$PM_t^B = \frac{\sum_{h=1}^{H}\left[(1-\beta)\frac{G}{H} + \beta\frac{1}{H}\frac{\sum_{g=1}^{G} N_{t+G+h-g}}{N_t}\right]}{\sum_{g=1}^{G}\left[\beta + (1-\beta)\frac{1}{H}\frac{\sum_{h=1}^{H} N_{t-G+g-h}}{N_t}\right]} \tag{25}$$

Table 3 illustrates this pension scheme for the same stylized demographic structure as was used for Table 2. In this example again 6 generations are affected by the one-time drop in population growth. The reduction is now highest for generation t, the first smaller generation and the reduction decreases for generations both preceding and succeeding this generation. As in case A the proportionality measure for a certain generation t depends on cohort sizes ranging from $N_{t-G-H+1}$ to $N_{t+G+H-1}$. We will discuss these patterns more extensively below.

`Insert Table 3 about here`

5 Comparison of Different Pension Schemes

The pension schemes that were presented in the last section differ along various dimensions. In this section we want to discuss these differences and deal with the relative merits of schemes A and B (for different values of α and β).

First we want to emphasize again that the one dimension along which the schemes are identical is fiscal sustainability. For both A and B it is true that the balanced budget condition (14) is fulfilled in every period. In addition to this, however, there are differences between the schemes that are related to fluctuations of the contribution rate and the pension level, to the characteristics of the proportionality measure and to aspects of practical implementation. We will deal with them in turn.

5.1 Fluctuations of τ, q and of PM

For a policy maker but also for the insured population it is important to know what implications a certain pension scheme has on the development of the contribution rates and the pension level over time. There are three types of fluctuations that are relevant: (i) Fluctuations of the average values $\bar{\tau}_t$ and $\bar{q}_t$ over time; (ii) Fluctuations in a certain time period t between different generations (i.e. differences in $\tau_{g,t}$ and $q_{h,t}$ for $g = 1, \ldots, G$ and $h = 1, \ldots, H$, respectively); (iii) Fluctuations of the rates over the lifetime of one specific generation (i.e. of $\tau_{g,t+g-1}$ and $q_{h,t+G+h-1}$ for $g = 1, \ldots, G$ and $h = 1, \ldots, H$, respectively). These types of fluctuations are of course related but together they provide one characterization of the different pension schemes. Related to this aspect one could also judge the quality of a specific pension scheme by investigating whether or under what conditions it exceeds certain "target values" (e.g., a minimum pension level or a maximum contribution rate).

We want to study these properties for simulated sequences of birth cohorts that are based on two demographic scenarios. In the first scenario (D1) we assume that the size of cohorts fluctuates around a constant mean of 100 and that the standard deviation is 10.[17] Thus under D1 we have: $N_t = 100 + \varepsilon_t$, where $\varepsilon_t \sim N(0, 100)$. This is of course a rather "conservative" (or "optimistic") scenario since in most countries the cohort size has shrunken over the last decades. The second demographic scenario (D2) is therefore modelled in a way that reflects the Austrian development (cf. Figure 1). In particular we estimate an AR(1) process for the Austrian cohort growth rate from 1955 to 2000. The estimated equation is then used as the data-generating process for the simulated series of cohort sizes. We get: $n_t = -0.00237 + 0.6 * n_{t-1} + u_t$, where $u_t \sim N(0, 0.000686)$. This scenario is also extreme since the negative expected value of the growth rate means that the cohort size will approach zero over time. Nevertheless it mirrors the real-world development over the last decades and it can be regarded as a second interesting reference case besides D1. The data presented below are based on 10000 observations from these simulated series.

`Insert Table 4 about here`

In Table 4 we report important descriptive statistics for the average contributions rate $\bar{\tau}_t$ and the average pension level $\bar{q}_t$ for the two demographic scenarios. Using this table and formulas (20), (21), (23) and (24) we can make the following observation.

[17] This corresponds to the standard deviation of the de-trended Austrian time series for cohort sizes from 1955 to 2000 (Mean=102236, SD=9728.7).

Observation 1 ***Fluctuations of*** $\bar{\tau}_t$ *and* $\bar{q}_t$.
(i) For $\alpha = 1$ *and* $\beta = 1$ *there is no variation in the average contribution rate* $\bar{\tau}$ *over time. The standard deviation of* $\bar{\tau}$ *increases as* α *and* β *get smaller.*
(ii) For $\alpha = 0$ *and* $\beta = 0$ *there is no variation in the average pension level* $\bar{q}$ *over time. The standard deviation of* $\bar{q}$ *increases as* α *and* β *get larger.*
(iii) At one point in time all cohorts pay the same contribution rate and receive the same pension level in pension scheme A but not in B (except for $\beta = 1$*)*
(iv) The amount of variation is similar in A and B under both demographic scenarios.
(v) The variation of tax rates and pension levels can get large. Under scenario D2 contribution rates can range from 6% to 53.6% (for A and $\alpha = 0$*) and 7.4% to 61.2% (for B and* $\beta = 0$*) and pension levels from 0.224 to 1.992 (for A and* $\alpha = 1$*) and 0.231 to 2.025 (for B and* $\beta = 1$*).*
(vi) If one wants to guarantee a maximum contribution rate of around 40% one must have that $\alpha \geq 0.5$, $\beta \geq 0.5$*. For a guaranteed minimum pension level of, e.g.,* $q \geq 0.4$ *(in order to prevent poverty) one must have that* $\alpha \leq 0.5$, $\beta \leq 0.5$.

It is obvious that the variation in $\bar{\tau}_t$ is largest for $\alpha = 0$ and $\beta = 0$ where the whole burden of adjustment is imposed on changes in the contribution rate. The opposite is true for $\alpha = 1$ and $\beta = 1$ where the adjustment is solely done by variations in q. Note that for pension scheme B the variation in the average magnitudes is not equal to the total variation, since here the rates and levels mighty also vary across cohorts at some point in time. The range of possible values for $\bar{\tau}_t$ and $\bar{q}_t$ is certainly extreme under scenario D2, while on the other hand the variation under D1 is quite likely to be too conservative.

If the quality of a pension system is primarily judged with respect to the variations it implies for the key parameters then observation 1 suggests a number of conclusions. The first is related to the question whether it is desirable to have different values for τ and q for different cohorts in some period. One could argue, e.g., that an age-based differentiation of contribution rates is politically and psychologically problematic since these rates are quite visible figures and people might have difficulties in understanding such a pattern. Furthermore, an age-based differentiation of contribution rates could have unwanted and unforeseeable consequences for hiring and firing behavior and one might prefer a pension scheme that does not have this property. Turning to the pension level one has to note that most real-world pension systems are in fact characterized by cross-generational differences in q at a certain point in time. Two persons X and Y that have identical working careers but are members of different generations mostly get different pension levels in the same calendar year. This is due to the definition of various adjustment and revaluation factors. The German point system is an exception in this respect since it leads to equal pensions for persons with the same number of points independent of the identity of their birth cohort. In general, people seem to be rather willing to accept changes in q than in τ (which could of course also be a consequence of the often quite intransparent pension calculations). Summing up we can conclude that the avoidance of age-based differentiation of contribution rates suggests the introduction of a pension scheme of type A or of system B with $\beta = 1$.

In addition to this there is, however, the crucial aspect of the variations in τ and q. As is shown in Table 4 and stated in parts (i) and (ii) of observation 1 the standard deviations of the average contribution rate and the average pension level are inversely related. It increases in α and β for $\bar{q}$ and decreases for $\bar{\tau}$. If one wants to prevent excessively strong fluctuations in any of the two variables then one should choose an in-between value around 1/2. Such a value is also likely to guarantee that the contribution rate stays below some maximum and the pension level above some minimum (poverty) level (cf. part (vi) of observation 1).

In fact such considerations seem to have been the dominating concern in Germany when the weight in the sustainability factor was set equal to $\alpha = 0.25$. In the Riester reform legislation from 2001 it was written down that the contribution rate should not rise above 20% by the year 2020 and above 22% by the year 2030. Furthermore it was stipulated that the government has to intervene if the pension level will fall below 67% of net earning. The sustainability factor and the precise weight were proposed with regard to these exogenous targets. "The weighting factor α implicitly sets the target contribution rate to be achieved. By setting α to 1/4, the Riester targets will be met." (Börsch-Supan, Reil-Held & Wilke, 2003, 16).

Despite the political and public emphasis on the variations in τ and q this is certainly not the only (and quite likely not even the most important) criterion one should take into consideration when choosing between different pension schemes. As a first extension of focus one can look at fluctuations in the proportionality measure itself. Accepting its role as an indicator of intergenerational fairness one could be inclined to prefer schemes that are characterized by small fluctuations in PM_t. We can look again at the properties of this measures for the two demographic scenarios D1 and D2 as shown in Table 5.

`Insert Table 5 about here`

Observation 2 ***Fluctuations of*** PM_t.
(i) The standard deviation of the proportionality measure PM_t *is lower for in-between values of* α *and* β*. For case A it is lowest for values around* $\alpha = 0.25$ *and for B for values around* $\beta = 0.5$*.*
(ii) The variations of PM can be large. Even for the in-between values they range — under scenario D2 — from 0.477 to 1.961 (for A and $\alpha = 0.25$*) and 0.43 to 2.196 (for B and* $\beta = 0.5$*).*

For the first demographic scenario the fluctuations in PM are again considerably lower (0.97 to 1.036 for $\alpha = 0.25$ and 0.97 to 1.033 for $\beta = 0.5$), although this is — as argued above — most likely an unreasonably stable scenario. Note that for the second demographic scenario with an expected downward trend in the size of birth cohorts also the mean (or expected value) of the proportionality measure is smaller than one.[18] The standard deviation of the proportionality measure is larger for $\alpha = 1$ and $\beta = 1$ than for

[18] In the simulation-based results of Table 5 it lies around 0.85. If we assume the same data-generating process (i.e the same downward trend) but without uncertainty then we get a value of 0.839.

$\alpha = 0$ and $\beta = 0$, since in this case only q is varying and there are fewer retired than active cohorts.

So considered from this angle one could again come to the conclusion that a value of α or β around 0.5 or maybe even below might be preferable. This could even be regarded as a justification for the German choice of $\alpha = 0.25$.

It is, however, not clear whether one should prefer the pension scheme with the smallest fluctuations of PM_t.[19] The principle should rather be that differences in PM between generations — should they occur — have a pattern that can be explained and defended by differences in their social and economic behavior. The crucial issue in our context is how differences in the cohort sizes and in reproductive behavior should be related to the proportionality measure and which of the pension schemes is closest to a framework that accords to generally accepted notions of intergenerational equity and fairness.

5.2 Properties of the Proportionality Measures

The question that was so far neglected in our discussion and that is also mostly neglected in the public debate is whether larger fluctuations in the proportionality measure (i.e. in the mix of contributions and benefits that different generations pay into and receive from a pension system) might perhaps be justifiable by reference to their reproductive or "birth-promoting" behavior. On an individual basis there exists in fact an ongoing debate where some economists argue that a PAYG pension system that does not include the number of children in the benefit formulas must be regarded as misconstructed. "In order to be able to consume in old age and enjoy a decent retirement life, a working generation has to save or to raise children who will later be able to pay them a pension. Or, to put it more bluntly, the working generation has to invest in real or in human capital. If it does not invest in either real or in human capital it will have to starve because nothing breeds nothing." (Sinn 2000, 24). On an individual level this would then amount to link the contributions and benefits to a person's number of children. "Instead of placing collective responsibility on a whole generation, the necessary pension cuts and the compensating new savings plan should be concentrated on the childless. Whoever has not raised children can be expected to take a pension cut of one half." (Sinn 2003, 1). This proposal is regarded as justified from the viewpoints of both the causation and the ability principle. The childless have caused the decline in cohort sizes and should bear the consequences while at the same time being in a position to be able to afford this since they do not face the costs of child rearing.

This view is, however, challenged on various grounds. Besides ideological reservations that are related to disreputable historical precursors of such proposals[20] it is often argued that it is hard to tell which persons are exactly to be held responsible for the size of a succeeding cohort. In modern societies it is not true that the natural parents are the

[19] In a different context Kifmann & Schindler (2000) argue for smoothed implicit tax rates. They note that it needs generation-specific contribution and replacement rates in order to achieve this which is similar in our framework for the proportionality measure.

[20] Cf. Barbier (2003).

only — or sometimes not even the main — sponsors of their offsprings. The welfare and tax systems know many channels by which the society as a whole shoulders part of the costs for the upbringing and the education of younger generations.[21] More generally the reproductive behavior is heavily influenced by the legal system and by common rules and norms. This system of incentives and disincentives is, however, shaped by the society at large (via electoral and political behavior and via general social activities) such that every individual and every generations bears at least some responsibility for the size of the succeeding generations.

Under a collective, cohort-based perspective[22] the question then becomes on which cohort sizes the proportionality measure of generation t should depend such that the underlying pension scheme can be regarded as a reasonable and equitable system. First one could argue that it should primarily reflect the size of the cohorts of generation t's prospective children, i.e. approximately of generations $t+20$ to $t+40$. In Austria 93.5% of all women and 87.5% of all men become parents when they are between 20 and 40. The negative financial consequences of a declining cohort size should be borne by the parent generation while — vice versa — the latter should also accredit gains from a possible increase. Alternatively and in addition one could, however, also take up the argument from above that each (politically and intellectually) mature cohort has an influence on the size of succeeding cohorts, either directly (through their reproductive behavior) or indirectly (through their voting behavior and their impact on the general social conditions). Thus it would be defendable that the proportionality measure depends on the cohort size of generations that are at least 20 years younger (or — strictly speaking — younger by 18 years, the voting age).

On the other hand it is more difficult to find convincing arguments why generation t should be held responsible for the size of their parents' and grandparents' generations. Time's arrow only points forwards and a cohort shouldn't be charged (or rewarded) for decisions that were taken before they were born. Certainly one could argue that such backward-looking dependency is part of intergenerational risk-sharing or that it is the compensation for other accomplishments or gifts that are valuable for today's population while they have shaped their parents' reproductive behavior. Admittedly these are difficult questions that cannot be addressed in a satisfying way in this paper. This would require at least a specification of the utility and cost of child-rearing and of the general tax and welfare system.[23] Nevertheless, as an additional piece of information one might want to know in as far a certain pension scheme implies a proportionality measure that is influenced by the size of succeeding vs. preceding cohorts.

In order to assess the dependence of the proportionality measure of generation t on the sizes of these particular groups of cohorts ($t+20$ to $t+40$; $t+20$ and younger) we have loglinearized the highly non-linear expressions for PM_t given in (22) and (25). This

[21] For Germany, e.g., it was calculated that the public sector pays about 40% of all related costs (Werding 1999).

[22] We will talk more about individual aspects in section 5.3.

[23] In our set-up we do not even specify what happens with the after-contribution wage $w_t(1-\tau)$ and under which conditions the new cohorts entering the labor market were raised.

allows us to write the proportionality measure in the following form (see appendix):

$$\ln PM_t = \sum_{s=-(G+H-1)}^{(G+H-1)} \delta_s \frac{N_{t+s} - N_t}{N_t} = \sum_{s=-(G+H-1)}^{(G+H-1)} \delta_s \ln \left(\frac{N_{t+s}}{N_t} \right) \tag{26}$$

The δ_s depend on G, H, α and β where the precise expressions are given in the appendix. They are in fact the elasticities of the proportionality measure with respect to the deviation of cohort size N_{t+s} from the size of the own cohort N_t. A $\delta_{20} = 0.05$ indicates, e.g., that the PM_t is 0.05% higher (ceteris paribus) if cohort N_{t+20} is 1% larger than cohort N_t. Figure 2 reports the pattern of these elasticities for pension scheme A and 5 different values for α when $G = 45$ and $H = 15$. Figure 3 does the same for scheme B and 5 different values for β. Two characteristics of these figures immediately catch the eye. First it is quite surprising how different the patterns look and second, it seems to be the case that a pension scheme is more "forward-looking" (i.e. more based on the size of succeeding cohorts) when α or β is larger.

`Insert Figures 2 and 3 about here`

In order to grasp the intuition behind the shape of the elasticities we want to focus first on the case with $\alpha = 1$. Since the contribution rate is fixed at $\widehat{\tau}$ for all times, generation t does not care about the total size of the labor force L_t and the total retirement population R_t while they are working. This changes, however, once they retire in period $t + 45$ (i.e., $t + G$) since then the relative sizes of L and R have an impact on their pension levels. When they reach retirement age there are 14 older generations that are already in pension. The more numerous these pensioner cohorts are the lower will be the relative pension and the proportionality measure of generation t. If the size of the pensioners aged 79 (cohort $t - 14$) is large then this is not so dramatic since they will only be around for one more year and will thus only depress the pension level for one year of the total retirement span. This is different, however, for the cohort that is only one year older (generation $t - 1$). They will be alive almost the whole pension period of generation t and will accordingly lower the pension level for 14 years. This is the reason why for case A the negative impact is largest for cohort size N_{t-1}.

Possible negative effects can of course be counterbalanced if the work force is large during generation t's retirement period. This is, however, a double-edged sword. If cohort $t + 1$ is large, e.g., this is of not much help since generation t will gain nothing from this large size until they are retired. Then, however, they have only 1 year to benefit from the large size of cohort $t + 1$ since a year later the latter will retire themselves from when on they will depress the relative pensions of generation t for 14 additional years. As shown in Figure 2 the net effect of a large cohort N_{t+1} is clearly negative. In fact this net effect stays negative up to cohort $t + 12$.[24] Generation t will prefer a situation where the large cohorts are the ones that contribute to the pension system for a long time while being at the

[24] For which cohort the balance between positive and negative effects of large succeeding cohorts turns positive depends on the relative sizes of G and H.

same time young enough such that they will only reach retirement age when generation t is already dead and gone. Cohorts $t+15$ to $t+44$ will pay contributions for the entire time when generation t is in pension and thus their impact on the proportionality measure is largest and identical as shown by the straight segment in Figure 2. For generations that are still younger ($t+45$ to $t+59$) this positive impact weakens since generation t will die before it can reap the full benefits of eventual larger cohort sizes. For this case there exist thus some incentives for generation t to leave enough offsprings since it will directly benefit from any self-induced baby boom.

We now turn to the opposite case with $\alpha = 0$ where the pension level is fixed at $\widehat{q}$. When generation t starts working there are 15 cohorts that are already in pension and will never pay any contribution while t is part of the pension scheme. Since their pension level is fixed the required contributions of generation t will increase with their size. This negative impact is largest for the cohort that just reached retirement age when generation t has started to work, i.e. for cohort $t-45$. All cohorts between $t-44$ and $t-14$ are partly contributors and partly pensioners while generation t is working. As in case A there is thus a trade-off involved. As shown in Figure 2 the net effect stays negative until cohort $t-11$, i.e. only for cohorts that are 31 or younger generation t will benefit from a large size. The "kink" at $t-14$ stems from the fact that this is the first cohort that generation t does not have to support through their whole retirement period since sooner or later t will itself reach pension age at which point the "burden" is left to the then working population. The best thing that can happen to generation t is when generation $t+1$ is large since this will attenuate any possible upward pressure on the contribution rate for almost all of the working life of t. Large cohorts that are younger than $t+1$ are also positive for t's proportionality measure although they will shoulder the common burden for a shorter period of time and thus the positive impact decreases with the youth of the cohort. As soon as generation t retires it does not have to care anymore about cohort sizes since its pension level is guaranteed. This explains why the elasticities for cohort sizes $t+45$ and above are zero. Note that in this case with $\alpha = 0$ generation t has less incentive to reproduce since the positive impact of cohort sizes $t+20$ to $t+40$ is much smaller than in the case with $\alpha = 1$. The intuition for the other cases follows a similar logic.

`Insert Table 6 about here`

In Table 6 we try to summarize the "hidden cohort dependence structure" of the ten pension schemes in a way that allows us to assess which one is more in line with alleged patterns of responsibility and fairness. In the first two rows of Table 6 we report the total sum of elasticities for the two prominent cohort groups mentioned above: the "children generations" (20-40, i.e. $t+20$ to $t+40$) and the "influenced generations" (20+, i.e. $t+20$ and younger). We see that the sum of elasticities of the 20-40 and the 20+ generations increases in α and β. In fact, for $\alpha = 1$ and $\beta = 1$ they are identical. In this case the numbers in the table indicate that if all cohorts that are between 20 and 40 years younger are 10% larger than generation t then its proportionality measure is — ceteris paribus —

4.67% higher. All else equal this means that the sum of their relative outcomes from the pension system will be 4.67% higher than the sum of their relative inputs (cf. (16)).

In the last two rows of Table 6 we report how important the elasticities of these special cohorts are for the total proportionality measure. Thereby we look at the sum of the elasticities divided by the total sum of (the absolute value of) elasticities.[25] Again we see that in pension schemes with a rather fixed contribution rate (high α and β) the proportionality measure is to a larger degree influenced by the cohorts for the size of which generation t bears some responsibility. In case B and $\beta = 1$, e.g., the relative weight of the children generation (20-40) is 46.7% and the one of the influenced generation (20+) is 73.3%. If one wants to strengthen the link between the PM and the size of cohorts for which a generation is in some way or another responsible then one must choose a high value for α in scheme A or — which is even more recommendable — a high value for β in scheme B.

At the same time, however, we see that none of the pension schemes is "perfect" in the sense that its proportionality measure depends only on these special generations. For example, all schemes except B with $\beta = 1$ imply a dependence of PM on preceding cohorts. As discussed above it is, however, hard to find convincing arguments why one should be held responsible for the size of cohorts that was determined before one was even born. This is a consequence of the general structure of schemes A and B. Of course one could think of additional schemes that lead to a balanced budget in every period and that avoid such "unwanted cohort dependencies". We have not been able, however, to find a balanced budget pension scheme that has, e.g., the property that $PM_t = f(N_{t+20}, N_{t+21,\ldots}, N_{t+40})$.[26]

To sum up, in this section we have tried to analyze how different pension schemes distribute the burden of demographic fluctuations between different cohorts and in which respect they can be regarded as reasonable and equitable. Even if we could not give a definite answer to the question which of the schemes is optimal with respect to intergenerational fairness we could at least show in a systematic way which dependencies between different cohorts are created by them and how they combine forward-looking and backward-looking elements. We have argued that schemes with a rather fixed contribution rate (high α and β) look more attractive from a perspective that focuses on issues of intergenerational fairness and responsibility. Finally we want to emphasize the obvious point that this "hidden cohort dependence structure" is present in pensions systems irrespective of whether one thinks about it or not. Every choice of a scheme and a weight is at the same time also a decision for a certain pattern of elasticities and cohort dependencies. The analysis of this section provides a framework that allows to take these issues into consideration when making the choice between different demographic adjust-

[25] That is, the measures "Share: 20-40", "Share: 20+" and "Share: 1+" are calculated as: $\frac{\sum_{s=20}^{40}|\delta_s|}{\sum_{s=-59}^{59}|\delta_s|}$, $\frac{\sum_{s=20}^{59}|\delta_s|}{\sum_{s=-59}^{59}|\delta_s|}$, $\frac{\sum_{s=1}^{59}|\delta_s|}{\sum_{s=-59}^{59}|\delta_s|}$.

[26] In an earlier version of this paper we have, e.g., presented two alternative schemes that have the properties that $PM_t = f(n_{t+1})$ and $PM_t = 1$, respectively. Although these schemes, especially the latter, might look attractive at first sight, they have other undesirable properties that make them impractible and unreasonable.

ment mechanisms. Since the public and academic discussion still focuses mostly on issues of fluctuations we think that such information can be quite useful.

5.3 Implementation

In this section we want to deal with two questions of implementation. First, how can a scheme that is regarded as intergenerationally fair be designed such that it also obeys principles of intragenerational fairness? Second, how can schemes A or B be implemented in existing PAYG systems or in notional defined contribution (NDC) systems.

So far we have looked at the issue of fairness from an intergenerational perspective. In the previous section we have analyzed how various pension schemes distribute the burden of demographic fluctuations among different cohorts and we have argued that it is justifiable to hold cohorts (at least partially) responsible for their own reproductive behavior. This collective responsibility masks, however, the fact that not every member of a cohort acts in the same way. While a considerable number of people has no descendants, some have three or more children. If these differences in reproductive behavior are neglected by a pension scheme and all members of a generation are treated alike then this could also be regarded as an unfair set-up. These considerations bring us back to the above-mentioned discussion whether *individual* pension benefits should be linked to a person's number of children.[27] In the following we will show that one cannot create a sustainable pension scheme by simply including the number of children into otherwise standard formulas for pension benefits. Furthermore we will discuss a method to adapt the collective equations (20), (21), (23) and (24) in a way that brings them more in line with notions of intragenerational fairness.

First we have to introduce some notation. Each member $i = 1, \ldots, N_t$ of generation t faces individual contribution rates $\tau^i_{g,t+g-1}$ and individual pension levels $q^i_{h,t+G+h-1}$ and has K^i_t children. It holds that:

$$\tau_{g,t} = \frac{1}{N_{t-g+1}} \sum_{i=1}^{N_{t-g+1}} \tau^i_{g,t} \tag{27}$$

$$q_{h,t} = \frac{1}{N_{t-G-h+1}} \sum_{i=1}^{N_{t-G-h+1}} q^i_{h,t} \tag{28}$$

Depending on the fertility age of the cohort members the number of children defines the size of the succeeding generations. For the sake of simplicity we will first assume that there is a fixed fertility age F that is identical for all (male and female) members of a

[27] One could of course also ask, whether the contribution rates should be linked to the number of children. This aspect, however, is less frequently mentioned in the literature. The reason is perhaps that the number of children is uncertain for a good part of the working life while it is practically determined for pensioners.

generation. Then we have that:[28]

$$N_{t+F} = \frac{1}{2}\sum_{i=1}^{N_t} K_t^i \tag{29}$$

The proposals of a "child-pension" now recommend that the benefit formula includes the number of children and that, e.g., childless people have to accept "a pension cut of one half". In the language of our model this amounts to an expression like:

$$q_{h,t}^i = \hat{q}\left[1 + \mu\left(\frac{K_{t-G-h+1}^i}{2} - 1\right)\right] \tag{30}$$

For $\mu = \frac{1}{2}$ we have in fact the result that a childless person ($K_{t-G-h+1}^i = 0$) receives $\frac{\hat{q}}{2}$, i.e. half the regular pension level, while a "fully reproducing individual" ($K_{t-G-h+1}^i = 2$) gets the benchmark pension level $\hat{q}$. Using (28) and (29) we can calculate the average pension level:

$$q_{h,t} = \hat{q}\left[1 + \mu\frac{N_{t-G-h+1+F} - N_{t-G-h+1}}{N_{t-G-h+1}}\right] \tag{31}$$

Contrasting this expression to (21) and (24) one sees that while scheme A determines $q_{h,t}$ by comparing $\bar{R}_t$ and $\bar{L}_t$ and scheme B by comparing $\bar{L}_t$ and $N_{t-G-h+1}$, this scheme is built on a comparison between the own size $N_{t-G-h+1}$ and the size of the cohort of the direct descendents $N_{t-G-h+1+F}$. We can now also calculate the average pension level in period t as (cf. (8)) $\bar{q}_t = \hat{q}\left[(1-\mu) + \mu\frac{1}{\bar{R}_t}\sum_{h=1}^{H} N_{t-G-h+1+F}\right]$. By inspection of this equation and of (14) it becomes evident that the use of an individual pension benefit formula like (30) does not automatically lead to a balanced budget. One can show, e.g., that even if the contribution rate formula incorporates the number of children this will not give rise to a financially balanced pension system. The basic reason for this is that the parents of generations $(t-G-F)$ to $(t-G)$ are not alive anymore in period t and so there is no one to take on the (financial) responsibility for their size. We can conclude that the simple use of child-related deductions and surcharges to a targeted pension level $\hat{q}$ does not automatically lead to a balanced budget.

There exists, however, a straightforward method that can be used to make individual adjustments to the collective formulas (21) and (24) and that increases the degree of intragenerational fairness. For the two cases $c \in \{A, B\}$ the individual formulas are:

$$\tau_{g,t}^{c,i} = \tau_{g,t}^{c} \tag{32}$$

$$q_{h,t}^{c,i} = \lambda_{t-G-h+1}^{i} q_{h,t}^{c}, \tag{33}$$

where $\lambda_{t-G-h+1}^i$ is the individual weight that determines which fraction of the collective pension level $q_{h,t}^A$ or $q_{h,t}^B$ an individual is awarded. This weight is related to the number of

[28]The $\frac{1}{2}$ in equation (29) stems from the fact that every child is counted twice in the summation formula.

children in the following way:

$$\lambda_t^i = \left(1 + \mu \frac{K_t^i - \bar{K}_t}{\bar{K}_t}\right) \tag{34}$$

where $\bar{K}_t$ is the average number of children of all cohort members: $\bar{K}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} K_t^i$. An individual that has less offsprings than the average cohort member will get a smaller fraction of $q_{h,t}^c$ than a person with many children. For the extreme case with $\mu = 1$ a childless person will not get any pension and will have to rely completely on savings in order to finance his or her retirement. Note that $\frac{1}{N_t}\sum_{i=1}^{N_t}\lambda_t^i = 1$ and so using (33) and (28) we can conclude that such an individual system has a balanced budget in every period just as the collective schemes A and B on which it is based. Using expressions (17) and (18) we can also derive that:

$$PM_t^i = \frac{\widetilde{q}_t^i}{\widetilde{\tau}_t^i} = \frac{\sum_{h=1}^{H} q_{h,t+G+h-1}^i}{\sum_{g=1}^{G} \tau_{g,t+g-1}^i} = \lambda_t^i \frac{\widetilde{q}_t}{\widetilde{\tau}_t} \tag{35}$$

This means that the individual proportionality measures are weighted with λ_t^i, meaning that within each cohort there is a positive correlation between the proportionality measure and the number of children. In this sense the use of formulas (32) and (33) leads to a situation that is arguably more in line with notions of intragenerational fairness than if all members are treated alike. At the same time the use of these formulas does not require information about the average and individual fertility ages etc., just the knowledge about the average number of children of a generation. A disadvantage of this individual scheme is, however, that the individual proportionality measures (35) are not completely in line with certain specific aspects of intragenerational fairness. It does not hold, e.g., that a "fully reproducing person" (i.e., one with two children) automatically has a unitary proportionality measure as one might want to require from a pension system. If, e.g., every member of cohort t has exactly two children while all cohorts before and after t have a smaller average number of descendants then the PM_t^i will be lower than 1 under all schemes A and B. We leave it as a topic of future research to investigate whether it is possible to find a pension scheme that has the property that $PM_t^i = 1$ for $K_t^i = 2$, $\forall t, i$ and that is characterized by a constantly balanced budget.

Next we want to abstain again from these intragenerational questions and ask ourselves, how the collective values for $\tau_{g,t}$ and $q_{h,t}$ can best be implemented in a real world pension system. Typically pension benefits are not calculated in a way that follow the formulas for $q_{h,t}^A$ and $q_{h,t}^B$. The pension level is not determined as a certain percentage q of *actual* average wages[29] but they are calculated in a "backward looking fashion". Normally the first pension is derived from the revalued wages (or contributions) of the worker and the following pensions are derived by adjusting this initial pension for changes in the cost of living (using, e.g., the rate of inflation or of wage growth). The revaluation and

[29] The exception is again Germany (and to a lesser degree France) where the point system implies this property.

adjustment factors thus play a crucial role in most countries' pension systems. This is even more important in "notional defined contribution" (NDC) systems, where the annual contributions to the PAYG system are credited to an individual (notional) account which increases in value over time according to some determined (notional) interest rate (which corresponds to the revaluation factor of classic PAYG systems). The question then is whether and how the formulas for (20), (21), (23) and (24) can be replicated in such "backward looking" calculative frameworks. Without going into too much detail here we want to stress two important points.

First one can show (cf. Lindbeck & Persson 2003, 86f.) that it is not possible to design a pension system that has (i) a fixed contribution rate, (ii) a fixed relative pension level and that (iii) uses either the growth of average wages or of the growth rate of the wage sum as indexation method and that leads to a balanced budget for a fluctuating demographic structure. Although this result is repeatedly mentioned in the literature it seems to be often forgotten in the political discussion. In fact it runs counter to the often heard claim that a revaluation with the growth rate of the wage sum will automatically make a PAYG pension system resistant to demographic fluctuations since this corresponds to its internal rate of return. This misstatement probably stems from the widespread use of two-period OLG models where the claim is in fact true.

It is clear from this result that if the cohort sizes fluctuate the traditional revaluation practices are not compatible with a sustainable PAYG system that has constant contribution rates and constant regulations for the determination of the pension level. In order to prevent financial crises one can either change the crucial parameters of the system from time to time in a discretionary manner (which was the common practice in many countries over the recent years) or one can try to introduce adjustment factors that explicitly account for these demographic gyrations. This brings us back to the pension schemes A and B and the question how these demographic adjustment factors can be implemented.

Here one has to note that it seems hard to frame schemes A and B with $\alpha < 1$, $\beta < 1$ as NDC systems. In these cases the contribution rate varies every period and thus an ever changing portion of earnings is credited to the notional account each period (cf. Valdés-Prieto 2000). It is difficult to see how a transparent notional interest rate regulation could be designed that would translate these indexed credits into the pension level $q_{1,t}$. Such a practice would mean that the value of the *accumulated* balances on the notional accounts will be changed every period in a way that is hard to understand by the insured persons and that do not resemble anymore the practices that are known from conventional savings accounts. In fact this seems to contradict one of the alleged advantages of a NDC system — increased transparency and predictability.

The cases with a fixed contribution rate ($\alpha = 1$, $\beta = 1$) seem to be better suited for a NDC system. It can be shown that there exist fairly simple and transparent frameworks that mimic traditional savings accounts and exactly lead to the formulas (21) and (24). Thereby the notional interest rate (or revaluation factor) is set equal to the growth rate of the wage sum and there is a one-time adjustment to the amount credited to the notional account that depends on a comparison of average and/or individual cohort sizes.

To sum up. We have shown in the second part of this section that, first, the commonly used indexation practices (with either average wage growth or the growth of the wage sum) are not suitable to keep a PAYG system with fixed τ and q but fluctuating N in fiscal balance. Some sort of demographic adjustment factors have to be incorporated into the system. In addition we have argued that schemes that have varying contribution rates ($\alpha < 1$, $\beta < 1$) are harder to reconcile with a NDC system than schemes with fixed contribution rates.

6 Conclusion

In this paper we have analyzed sustainable pension schemes that are capable of dealing with fluctuations in the fertility rate. For each of the schemes we have presented formulas that state how the contribution rates and/or the pension levels have to be adjusted in order to have a balanced budget in every period.

Depending on the weights the schemes have different properties concerning the fluctuations of the crucial parameters (both over time and across generations), concerning the intergenerational burden sharing of demographic changes and the ease with which they can be implemented in a transparent way. Viewed solely from the perspective of intergenerational smoothing one should opt for in-between values of the weights. Thereby it is also most likely that a certain maximum contribution rate or a minimum pension level target will be fulfilled. Furthermore a weight between 0.25 and 0.5 will also lead to the smallest fluctuations in the proportionality measure—an indicator we have proposed to assess the implications of different schemes on the intergenerational distribution of the demographic burden.

The drawback of such in-between values is, however, that it is not clear that the intergenerational distribution implied by these schemes can be regarded as intergenerationally fair. We have shown that the proportionality measures of schemes with rather fixed pension levels are more strongly "backward looking", indicating that people are obliged to shoulder the burden of changes in the size of cohorts that have been determined before they were even born or before they were part of the electorate, the labor force and thus of the potential parent generations. Schemes with fixed contribution rates on the other hand are more forward-looking and one could thus say that they are preferable as far as intergenerational fairness and as far as (implicit) incentives for reproduction are concerned.

The latter property holds, however, only under a collective perspective, i.e. for a cohort as a whole. We have shown that there exists a modification to the collective formulas that considers individual reproductive behavior and that is more in line with general notions of intragenerational fairness. Finally we have shown that schemes with fixed contribution rates are more easily framed in NDC systems than systems with fluctuating rates and the latter should better be implemented in a traditional manner (as, e.g., the German point system).

As this paper has shown the choice of a pension scheme and an appropriate weight is a difficult and multi-faceted issue. Certainly we cannot give a definite answer or suggestion since this will depend on the political and social preferences, the specific national situation etc. The purpose of the paper was rather to argue that these decisions should not only be taken with regard to sustainability, intertemporal smoothing or the adherence to some exogenously given target, but also with regard to questions of intergenerational fairness and responsibility. Since the eventual introduction of automatic demographic adjustment factors is high on the reform agenda in various countries we believe that these considerations will become even more important over the coming years.

7 Appendix

Case A: Using (20), (21) and the definitions (7) and (8) we can calculate that: $\bar{\tau}_t = \hat{\tau}\frac{\bar{L}_t+(1-\alpha)(\bar{R}_t-\bar{L}_t)}{\bar{L}_t}$ and $\bar{q}_t = \hat{q}\frac{\bar{R}_t+\alpha(\bar{L}_t-\bar{R}_t)}{\bar{R}_t}$. Inserting these values into the balanced budget condition (14) and using (15) we get: $\alpha\bar{L}_t + (1-\alpha)\bar{R}_t = \alpha\bar{L}_t + (1-\alpha)\bar{R}_t$ and thus the budget is balanced in every period. For the sum of relative inputs and outcomes we get:

$$\widetilde{\tau}_t^A = \widehat{\tau}\sum_{g=1}^{G}\left[\alpha + (1-\alpha)\frac{\bar{R}_{t+g-1}}{\bar{L}_{t+g-1}}\right] = \widehat{\tau}\sum_{g=1}^{G}\left[\alpha + (1-\alpha)\frac{\frac{1}{H}\sum_{h=1}^{H}N_{t-G+g-h}}{\frac{1}{G}\sum_{j=1}^{G}N_{t+g-j}}\right] \tag{A1}$$

$$\widetilde{q}_t^A = \hat{q}\sum_{h=1}^{H}\left[(1-\alpha) + \alpha\frac{\bar{L}_{t+G+h-1}}{\bar{R}_{t+G+h-1}}\right] = \hat{q}\sum_{h=1}^{H}\left[(1-\alpha) + \alpha\frac{\frac{1}{G}\sum_{g=1}^{G}N_{t+G+h-g}}{\frac{1}{H}\sum_{j=1}^{H}N_{t+h-j}}\right] \tag{A2}$$

From this equation (22) for the proportionality measure PM_t^A follows.
Case B: Similarly we can calculate: $\bar{\tau}_t = \hat{\tau}\frac{\beta L_t+(1-\beta)R_t G}{L_t}$ and $\bar{q}_t = \hat{q}\frac{(1-\beta)R_t+\beta L_t H}{R_t}$. Inserting these values into the balanced budget condition (14) and using again (15) we get: $\beta\bar{L}_t + (1-\beta)\bar{R}_t = \beta\bar{L}_t + (1-\beta)\bar{R}_t$ and thus the budget is again balanced in every period. For $\widetilde{\tau}_t^B$ and $\widetilde{q}_t^B$ we get:

$$\widetilde{\tau}_t^B = \widehat{\tau}\sum_{g=1}^{G}\left[\beta + (1-\beta)\frac{\bar{R}_{t+g-1}}{N_t}\right] = \widehat{\tau}\sum_{g=1}^{G}\left[\beta + (1-\beta)\frac{\frac{1}{H}\sum_{h=1}^{H}N_{t-G+g-h}}{N_t}\right] \tag{A3}$$

$$\widetilde{q}_t^B = \hat{q}\sum_{h=1}^{H}\left[(1-\beta) + \beta\frac{\bar{L}_{t+G+h-1}}{N_t}\right] = \hat{q}\sum_{h=1}^{H}\left[(1-\beta) + \beta\frac{\frac{1}{G}\sum_{g=1}^{G}N_{t+G+h-g}}{N_t}\right] \tag{A4}$$

Log-linearization of the proportionality measure:

For both cases A and B the proportionality measure is a non-linear function of the various cohort sizes, i.e.

$$PM_t = f(N_{t-G-H+1}, N_{t-G-H+2}, \ldots, N_{t+G+H-2}, N_{t+G+H-1})$$

We can take the logarithm of this expression and then linearize $\ln PM_t$ at $N_s = N_t$ for $(t-G-H+1) \leq s \leq (t+G+H-1)$. This leads to the following expression:

$$\ln PM_t \approx \sum_{s=-(G+H-1)}^{(G+H-1)}\delta_s\frac{N_{t+s}-N_t}{N_t} = \sum_{s=-(G+H-1)}^{(G+H-1)}\delta_s\ln\left(\frac{N_{t+s}}{N_t}\right) \tag{A5}$$

The δ_s are thus the elasticities of the proportionality measure with respect to the deviation of cohort size N_{t+s} from the size of the own cohort N_t. For case A these can be calculated as:

For	δ_s
$-(G+H-1) \leq s < -G$	$-(1-\alpha)\frac{G+H+s}{GH}$
$-G \leq s < -H$	$-(1-\alpha)\frac{1}{G} + (1-\alpha)\frac{G+s}{G^2}$
$-H \leq s \leq -1$	$(1-\alpha)\frac{s}{GH} + (1-\alpha)\frac{G+s}{G^2} - \alpha\frac{H+s}{H^2}$
$1 \leq s < H$	$\alpha\frac{s}{GH} + (1-\alpha)\frac{G-s}{G^2} - \alpha\frac{H-s}{H^2}$
$H \leq s < G$	$\alpha\frac{1}{G} + (1-\alpha)\frac{G-s}{G^2}$
$G \leq s \leq (G+H-1)$	$\alpha\frac{G+H-s}{GH}$

For case B they come out as:

For	δ_s
$-(G+H-1) \leq s < -G$	$-(1-\beta)\frac{G+H+s}{GH}$
$-G \leq s < -H$	$-(1-\beta)\frac{1}{G}$
$-H \leq s \leq -1$	$(1-\beta)\frac{s}{GH}$
$1 \leq s < H$	$\beta\frac{s}{GH}$
$H \leq s < G$	$\beta\frac{1}{G}$
$G \leq s \leq (G+H-1)$	$\beta\frac{G+H-s}{GH}$

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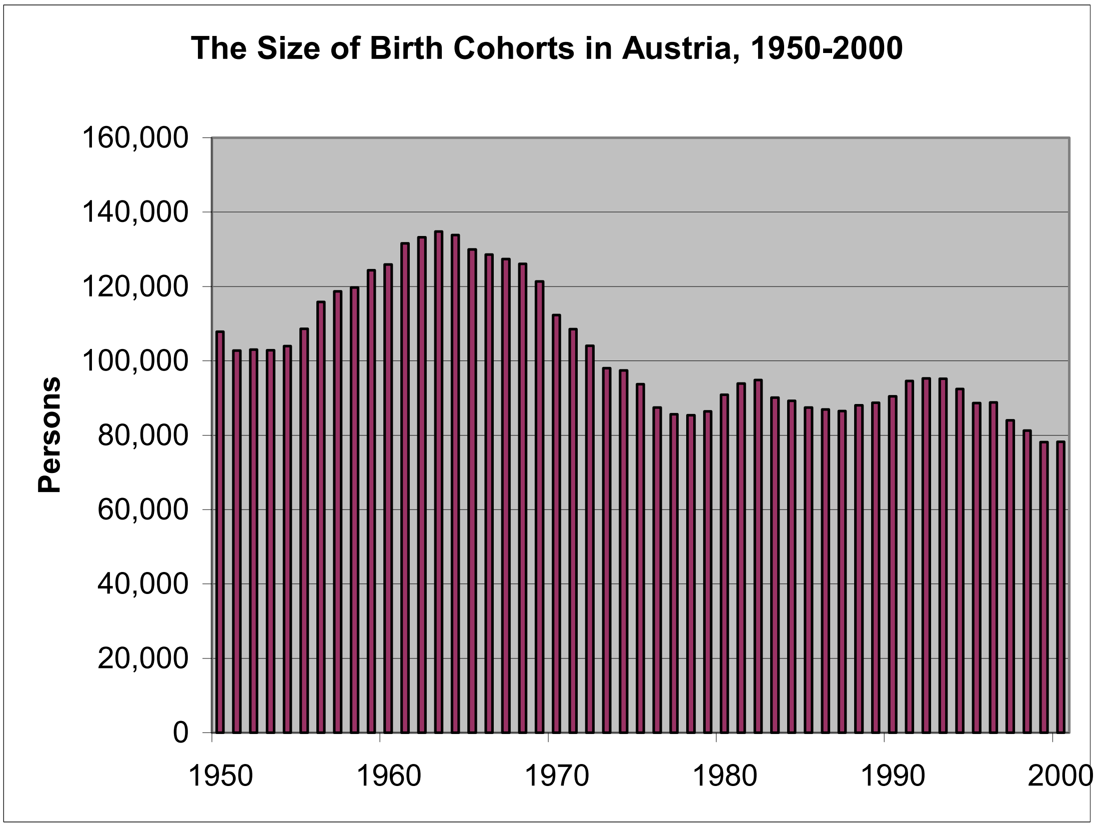
Figure 1



Source: Statistik Austria

Table 1

A Simple Example for a PAYG Pension System

Generation ↓		← Time →								Sum of		Ratio
		$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$	Rel. Inputs	Rel. Output	Rel. Out/ Rel. In
⋮	⋮	⋮	⋮	⋮						…	…	…
N_{t-3}:	100	0.2	0.2	0.2	0.6					0.6	0.6	1
N_{t-2}:	100		0.2	0.2	0.2	0.6				0.6	0.6	1
N_{t-1}:	100			0.2	0.1	0.2	0.6			0.5	0.6	1.2
N_t:	100				0.3	0.2	0.2	0.6		0.7	0.6	0.86
N_{t+1}:	100					0.2	0.2	0.2	0.6	0.6	0.6	1
N_{t+2}:	100						0.2	0.2	0.2	0.6	…	…
N_{t+3}:	100							0.2	0.2	…	…	…
⋮	⋮								0.2	…	…	…
Budget in Year *y*:					0	0	0	0	0			
EX_y / w_y:					60	60	60	60	60			
IN_y / w_y:					30+10+20	20+20+20	20+20+20	20+20+20	20+20+20			

Contribution Rate (τ) in Red (Light Shading); Relative Pension Level (q) in Blue (Dark Shading).

Table 2

A One-Time Drop in Cohort Size: Case A (α = 0.5)

Generation ↓		← Time →								Sum of		Ratio
		$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$	Rel. Inputs	Rel. Output	Rel. Out/ Rel. In
⋮	⋮	⋮	⋮	⋮						…	…	(1.00)
N_{t-3}:	100	0.2	0.2	0.2	0.55					0.60	0.55	0.92
N_{t-2}:	100		0.2	0.2	0.22	0.50				0.62	0.50	0.81
N_{t-1}:	100			0.2	0.22	0.25	0.45			0.67	0.45	0.67
N_t:	50				0.22	0.25	0.30	0.6		0.77	0.60	0.78
N_{t+1}:	50					0.25	0.30	0.2	0.6	0.75	0.60	0.80
N_{t+2}:	50						0.30	0.2	0.2	0.70	0.60	0.86
N_{t+3}:	50							0.2	0.2	0.60	0.60	1.00
⋮	⋮								0.2	…	…	…
Budget in Year *y*:					0	0	0	0	0			
EX_y / w_y:					55	50	45	30	30			
IN_y / w_y:					11+22+22	12.5+12.5+25	15+15+15	10+10+10	10+10+10			

Contribution Rate (τ) in Red (Light Shading); Relative Pension Level (q) in Blue (Dark Shading).

Table 3

A One-Time Drop in Cohort Size: Case B (β = 0.5)

Generation ↓		← Time →								Sum of		Ratio
		$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$	Rel. Inputs	Rel. Output	Rel. Out/ Rel. In
⋮	⋮	⋮	⋮	⋮						…	…	(1.00)
N_{t-3}:	100	0.2	0.2	0.2	0.55					0.6	0.55	0.92
N_{t-2}:	100		0.2	0.2	0.2	0.50				0.6	0.5	0.83
N_{t-1}:	100			0.2	0.2	0.2	0.45			0.6	0.45	0.75
N_{t}:	50				0.3	0.3	0.3	0.6		0.9	0.6	0.67
N_{t+1}:	50					0.3	0.3	0.2	0.6	0.8	0.6	0.75
N_{t+2}:	50						0.3	0.2	0.2	0.7	…	(0.86)
N_{t+3}:	50							0.2	0.2	…	…	…
⋮	⋮								0.2	…	…	…
Budget in Year y:					0	0	0	0	0			
EX_y / w_y:					55	50	45	30	30			
IN_y / w_y:					15+20+20	15+15+20	15+15+15	10+10+10	10+10+10			

Contribution Rate (τ) in Red (Light Shading); Relative Pension Level (q) in Blue (Dark Shading).

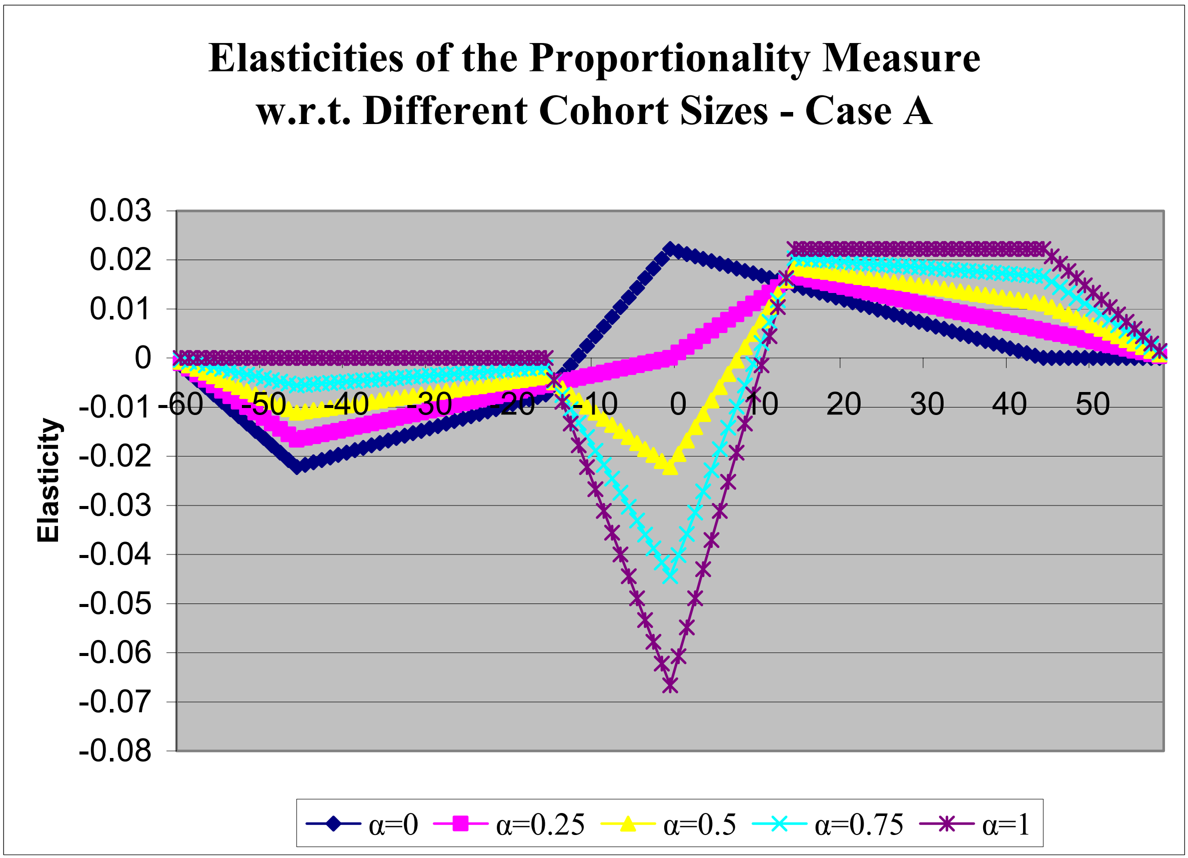
Table 6

Summary of Elasticties for Special Groups of Cohorts

	Case A Value for α					Case B Value for β				
	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
Sum: 20-40	0.156	0.233	0.311	0.389	0.467	0.000	0.117	0.233	0.350	0.467
Sum: 20+	0.160	0.304	0.447	0.590	0.733	0.000	0.189	1.000	2.333	0.875
Share: 20-40	12.7%	23.3%	26.8%	27.5%	27.7%	0.0%	11.7%	23.3%	35.0%	46.7%
Share: 20+	13.1%	30.4%	38.5%	41.7%	43.5%	0.0%	18.3%	36.7%	55.0%	73.3%
Share: 1+	39.8%	50.0%	57.9%	66.2%	72.3%	0.0%	25.0%	50.0%	75.0%	100%

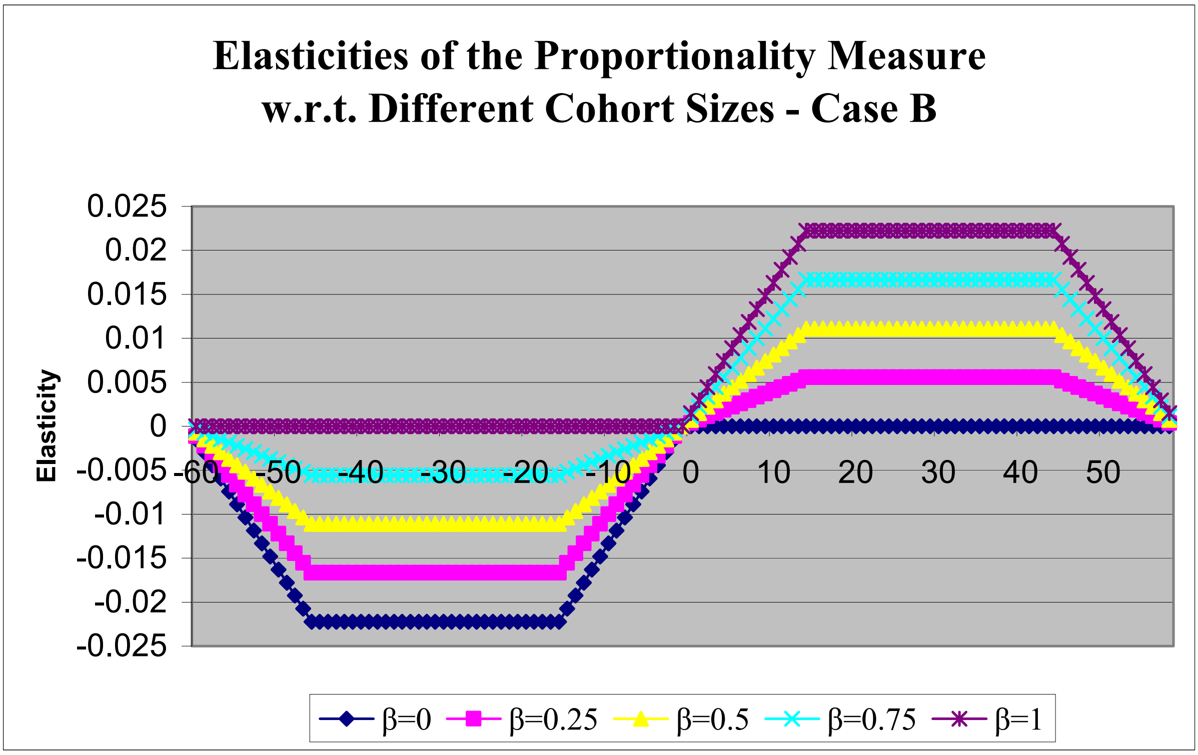
Note: The numbers in the first two rows gives the sum of the elasticities δ_s for (s=20, … , 40) and (s=20, … , 59), respectively. The last two rows contain *relative* measures (see FN 25).

Figure 2



Note: The lines give the elasticity of PM_t with respect to the deviation of the size N_{t+s} of cohort (*t+s*) from its own size N_t. The elasticity for *s=0* is not meaningful since the deviation of N_t from itself is obviously always zero.

Figure 3



Note: See Figure 2.

Table 4

(A) Fluctuations of $\overline{\tau}_t$

	Demographic Scenario 1 (Constant average cohort size)									
	Case A					*Case B*				
$\overline{\tau}_t$ ↓	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	0.200	0.200	0.200	0.200	0.200	0.202	0.202	0.201	0.201	0.200
SD	0.006	0.004	0.003	0.001	0.000	0.006	0.005	0.003	0.002	0.000
Min – Max	0.185-0.221	0.18-0.216	0.19-0.211	0.195-0.205	0.2-0.2	0.182-0.223	0.186-0.217	0.191-0.212	0.195-0.206	0.2-0.2

	Demographic Scenario 2 (Decreasing average cohort size)									
	Case A					*Case B*				
$\overline{\tau}_t$ ↓	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	0.250	0.237	0.225	0.212	0.200	0.260	0.245	0.230	0.215	0.200
SD	0.069	0.052	0.034	0.017	0.000	0.075	0.056	0.037	0.019	0.000
Min – Max	0.06-0.536	0.095-0.452	0.13-0.368	0.165-0.284	0.2-0.2	0.074-0.612	0.105-0.509	0.137-0.406	0.168-0.303	0.2-0.2

(B) Fluctuations of $\overline{q}_t$

	Demographic Scenario 1 (Constant average cohort size)									
	Case A					*Case B*				
$\overline{q}_t$ ↓	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	0.600	0.600	0.600	0.600	0.600	0.600	0.602	0.603	0.605	0.606
SD	0.000	0.004	0.009	0.013	0.018	0.000	0.005	0.009	0.014	0.018
Min – Max	0.6-0.6	0.586-0.617	0.571-0.634	0.557-0.651	0.543-0.668	0.6-0.6	0.587-0.618	0.573-0.637	0.56-0.655	0.547-0.673

Table 4 (cont.)

	Demographic Scenario 2 (Decreasing average cohort size)									
	Case A					*Case B*				
$\overline{q}_t \downarrow$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	0.600	0.581	0.562	0.543	0.523	0.600	0.582	0.564	0.546	0.528
SD	0.000	0.043	0.087	0.130	0.173	0.000	0.044	0.087	0.131	0.174
Min – Max	0.6-0.6	0.506-0.948	0.412-1.296	0.318-1.644	0.224-1.992	0.6-0.6	0.508-0.956	0.415-1.312	0.323-1.669	0.231-2.025

Table 5

Fluctuations of PM_t

	Demographic Scenario 1 (Constant average cohort size)									
	Case A					*Case B*				
$PM_t \downarrow$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	1.000	1.000	1.000	1.000	1.001	1.000	0.999	1.000	1.004	1.010
SD	0.014	0.011	0.012	0.017	0.024	0.100	0.051	0.010	0.052	0.104
Min – Max	0.963-1.039	0.97-1.036	0.961-1.044	0.941-1.061	0.921-1.083	0.601-1.452	0.77-1.205	0.97-1.033	0.836-1.286	0.694-1.667

	Demographic Scenario 2 (Decreasing average cohort size)									
	Case A					*Case B*				
$PM_t \downarrow$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 1$
Mean	0.845	0.847	0.851	0.860	0.872	0.848	0.839	0.841	0.853	0.880
SD	0.214	0.193	0.200	0.231	0.279	0.289	0.218	0.197	0.227	0.299
Min – Max	0.45-2.517	0.477-1.961	0.501-2.306	0.483-2.747	0.386-3.196	0.346-3.323	0.379-2.014	0.43-2.196	0.485-2.713	0.366-3.576

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1) vergriffen (out of print)
2) In abgeänderter Form erschienen in Berichte und Studien Nr. 4/1990, S 74 ff
3) In abgeänderter Form erschienen in Berichte und Studien Nr. 4/1991, S 44 ff
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